The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2011-2012

MECHANICS OF SOLIDS 2

Time allowed ONE Hour and THIRTY Minutes

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer THREE questions

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

ADDITIONAL MATERIAL: Graph Paper

INFORMATION FOR INVIGILATORS:

1. The centroid of area, G, of the cross-section shown in Fig. Q1 is positioned 11.67 mm from the left hand edge and 41.67 mm from the bottom edge of the section, as shown. The section's principal second moments of area are:

$I_p = 19.25 \times 10^5$ mm^4

$I_q = 1.65 \times 10^5$ mm^4

The angle of the principal p-axis is 12.3° anticlockwise from the x-axis, as shown.

The section is subjected to a bending moment M = 1000 Nm acting in a direction 45° anticlockwise from the x-axis, as shown.

Determine:

(a) The orientation angle of the neutral axis with respect to the x-axis (show on a sketch of the cross-section). [13 marks]

(b) the position and magnitude of the maximum bending stress and state whether it is tensile or compressive. [20 marks]

The co-ordinate transformation equations for a set of axes, p-q, angled θ anticlockwise to the x-y axes, are:

$p = x\cos\theta + y\sin\theta$

$q = -x\sin\theta + y\cos\theta$

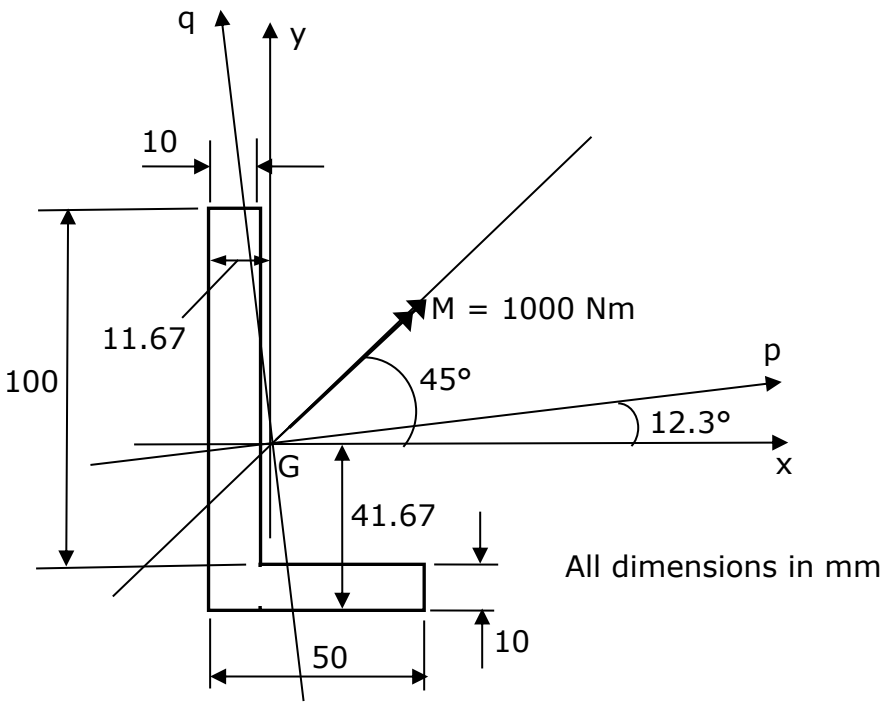

Fig. Q1

2. The support post for a street sign is subject to service conditions which create a bending load of 5 kN and a torsional load of 10 kNm at the top of the post, as shown in Fig. Q2. The post is made from a fibre reinforced composite (E = 40 GPa, G = 15 GPa), is 3 m high and has a hollow circular cross-section of outside diameter 220 mm and inside diameter 210 mm.

(a) Calculate the total strain energy (bending + torsional) stored in the post under this loading. [17 marks]

(b) Determine the deflection of the top of the post in the direction of the applied bending load. [8 marks]

(c) Determine the angular rotation of the top of the post. [8 marks]

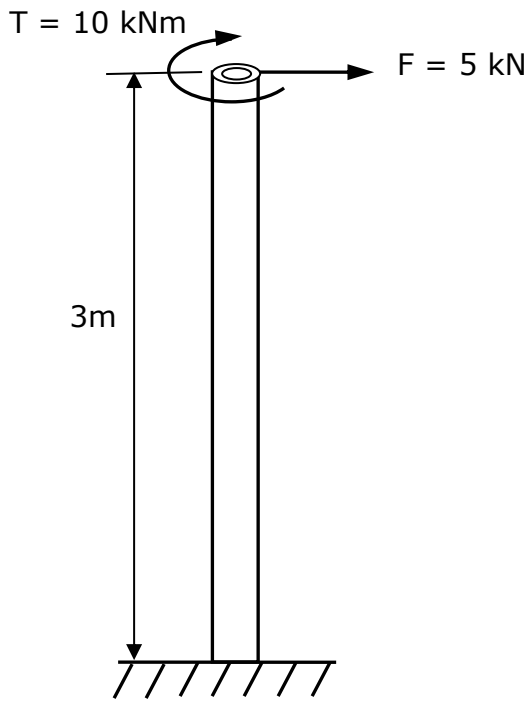

Fig. Q2

3. A steel disc, of internal diameter 200 mm and external diameter 1 m, rotates at an angular velocity of 5000 rpm.

The radial stress and circumferential stress terms are given by:

$$\sigma_r = A - \frac{B}{r^2} - \frac{\rho w^2 (3+\nu) r^2}{8}$$

and

$$\sigma_\theta = A + \frac{B}{r^2} - \frac{\rho w^2 (1+3\nu) r^2}{8}$$

Assume: ρ = 7850 kg/m^3 and ν = 0.3

Plot the circumferential and radial stress distributions in the disc. [33 marks]

4. A pin-jointed structure, ABC, shown in Figure Q4, is subjected to a vertical load W at joint C. The struts, BC and AC, in the structure, are made from solid circular bar of diameter *d*, and the material has the modulus of elasticity *E*.

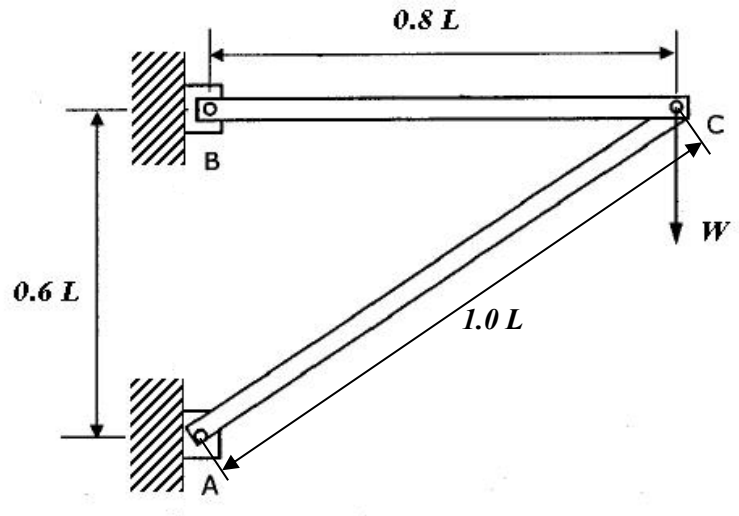

Fig. Q4

(a) State the equation for buckling load of a pinned-pinned strut in terms of the modulus of elasticity *E*, the second moment of area *I*, and the length of the strut *L*. [4 marks]

(b) Determine the maximum capacity of the structure ABC carrying the vertical load W in terms of *E*, *d*, and *L*. [18 marks]

(c) If the material of the strut AC is changed to another material with a modulus of elasticity *E*/3, but the length is maintained the same, determine the factor by which the diameter of the strut needs to be enlarged such that the structure still has the same capacity as before. [11 marks]

5. A pin-jointed framework, consisting of two truss members, is subjected to an external load *F* as shown in Figure Q5. Each member of the structure has the length *L*, cross sectional area A and modulus of elasticity *E*.

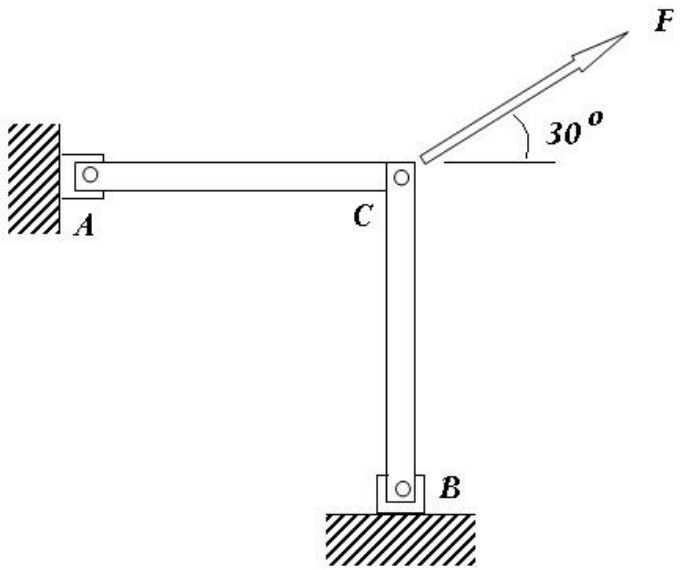

Fig. Q5

(a) Construct the stiffness matrix of the structure. [13 marks]

(b) Using the stiffness matrix, determine expressions for:

i. the horizontal and vertical displacements of point C [10 marks]

ii. the reactions on the supports A & B [10 marks]

The global stiffness matrix of a truss element is:

$$[k_e]_G = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

End